Commentary on the habilitation thesis

Riccati methods for half-linear differential equations

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The form of the thesis is a commentary on a collection of scholarly papers devoted to studies of the second-order half-linear differential equations [1, 2, 3, 4, 5, 6, 7, 8]. In the body of the thesis, their results are summarized and commented, and printouts of the full versions are enclosed at the end in the form of an attachment. We are interested mainly in the qualitative theory and asymptotic properties of studied equations, focusing on the applicability of the so-called modified Riccati technique. With its use, it is possible to investigate especially conditionally oscillatory equations, which occur at the threshold between oscillation and nonoscillation, and their perturbations. After the introductory part presenting the historical background and basic methods, the results providing concrete asymptotic formulas are commented. The next sections sum up our results in the form of various oscillation and nonoscillation criteria for ordinary and neutral half-linear equations. In the end, the numerical approach to finding approximate solutions of half-linear Euler-type equations, which makes use of the differential transformation method, is introduced. The thesis is concluded with a list of possible future directions of research.

The papers [1, 2, 3, 7, 8] were created in collaboration with coauthors. All the coauthors contributed equally to these works and the author participated in their preparation at every stage including research conceptualization, main investigation, and writing. The author's contribution in these papers is 50%.

References

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