MUNI

Annex No. 10 to the MU Directive on Habilitation Procedures and Professor Appointment Procedures

HABILITATION THESIS REVIEWER'S REPORT

Masaryk University	
Applicant	Mgr. Zuzana Pátíková, Ph.D.
Habilitation thesis	Riccati methods for half-linear differential equations
Reviewer	doc. Mgr. Petr Hasil, Ph.D.
Reviewer's home unit, institution	Masaryk University Faculty of Science Department of Mathematics and Statistics

The reviewed habilitation thesis of Zuzana Pátíková has the form of a commentary together with a collection of eight scientific papers written by herself alone (3 of them) or with a co-author. One of those papers was published in 2010, the rest of them during the years 2017-2022. I should note that the last paper (reference number [112]) was in the review process at the time of submission of the thesis and it is successfully published (with some minor changes) at the time of writing of this report in Journal of Computational Science, Volume 59, article number 101564.

The main part of the thesis, preceding the attachment with the mentioned papers, is divided into five chapters (plus Preface, Conclusion, and References). The first (introductory) chapter is devoted to the description of the background of the theory, both historical and methodological with the attention paid to Riccati method (and Reid's Roundabout theorem).

For the purpose of the overview, I sum up the main points of the chapters 2-5. In the second chapter, the author describes asymptoticity of non-oscillatory solutions presented in papers [109, 110]. The asymptotic formulas are described in terms of slowly and regularly varying functions (in the sense of Karamata). The third chapter deals with (non-)oscillation criteria, the results come from papers [65, 66, 111] and I should emphasize that there are given results concerning equations with perturbed potential as well. The fourth chapter describes the variant of Riccati technique for neutral half-linear equations and presents the results of [67] obtained via this method. The last chapter is quite different from the previous ones which is given by its focus on the numerical approach. It contains a description of results published in [112,114]. A differential transform algorithm is proposed. It is developed for finding numerical solutions of initial value problems even for delayed equations with nonconstant delays. As this kind of methods is based on Taylor's method it results to easy-to-use (in a certain sense) methods and I see a great potential in this direction of research, because it is one of the ways how it is possible to estimate the behaviour of very complex equations and models before a deeper study of their behaviour.

Finally, in the section Conclusion, the author summarizes some possible ways of extension and continuation of the research. In my opinion, the list of open problems presented there is far from completeness (which is not meant to be a negative) and the presented research has a perspective for a future development.

Regarding the technical side of the work, the language is adequate, the presentation of the results is clear, and the thesis is well organized. I consider the commentary well written and the results contained in the habilitation thesis declare the capability of the author to conduct an independent research. I recommend accepting the reviewed thesis as the scientific written part of the process of habilitation.

Reviewer's questions for the habilitation thesis defence

1. The theory of difference equations contains analogies to many tools and methods used in the differential case and it is widely used in applications and very useful. Are there discrete versions of described results, do you intend to proceed in your research in this direction or are there some obstacles that make this way of research hard and/or impossible?
2. Is it possible to quantify the computational complexity of the presented differential transformation method?

Conclusion

The habilitation thesis entitled "Riccati methods for half-linear differential equations" by Zuzana Pátíková **fulfils** requirements expected of a habilitation thesis in the field of Mathematics – Mathematical Analysis.

Date: February 14, 2022 Signature: