

## COMMENTARY TO HABILITATION THESIS<sup>1</sup>

### Qualitative theory of fractional differential systems with time delay

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Fractional calculus, an area dealing with derivatives of non-integer orders, has gained recognition for its linear descriptions of complex systems characterized by nonlocal or memory-based behaviour. Moreover, the ability to continuously transition between derivative orders reveals previously unseen connections and enables the study of various phenomena.

In particular, the area of analysis of fractional delay differential equations (FDDEs), a class of mathematical models involving fractional derivatives and time delays representing inherent lags in the system, proved to be quite useful as it deals with prediction and control of the behaviour of such systems occurring from physics and biology to engineering and finance. The literature on this matter is quite extensive and started developing mainly in the last twenty years.

This thesis is based on my seven selected papers [1-7] from 2016-2023 devoted to the subject of qualitative analysis of FDDEs. It summarizes their main original results such as stability and oscillatory conditions, often in non-improvable form, asymptotics of bounded and unbounded solutions. In particular, it explores the relationships between derivative order and the stability regions for the system's coefficients. Moreover, it provides commentary on the key proofs and employed techniques built on ideas of D-decomposition method, complex contour integrals, Jordan matrix theory, special functions theory, asymptotic estimates, convolution formulas, curve reparametrization etc.

In all the papers [1-7] I contributed in all areas relevant to mathematical research. My contributions to the results correspond to the % in the manuscript category and individual accents are detailed below.

[1]<sup>2</sup> Čermák, J., Kisela, T., Nechvátal, L. The Lambert function method in qualitative analysis of fractional delay differential equations. *Fractional Calculus and Applied Analysis* 26, 1545-1565 (2023). IF: 3.000, Q1

*I was primarily focused on proofs and visualization, involved also in computational work and validation of achieved results.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	33 %	10 %

<sup>1</sup> The commentary must correspond to standard expectations in the field and must include a brief characteristic of the investigated matter, objectives of the work, employed methodologies, obtained results and, in case of co-authored works, a passage characterising the applicant's contribution in terms of both quality and content.

<sup>2</sup> Bibliographic record of a published scientific result, which is part of the habilitation thesis.

**[2]** Čermák, J., Kisela, T. Stabilization and destabilization of fractional oscillators via a delayed feedback control. Communications in Nonlinear Science and Numerical Simulation 117, 1-16 (2023). IF: 3.900, Q1

*I was primarily focused on conceptualization, analysis and proofs, visualization and writing, participated in editing and problem formulation, involved also in computation and validation.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	50 %	70 %

**[3]** Kisela, T. On stability of delayed differential systems of arbitrary non-integer order. Mathematics for Applications 9, 31-42 (2020).

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	100 %	100 %

**[4]** Čermák, J., Kisela, T. Oscillatory and asymptotic properties of fractional delay differential equations. Electronic Journal of Differential Equations 2019, 1-15 (2019). IF: 0.820, Q2

*I was primarily focused on conceptualization, problem formulation, analysis and proofs, participated in visualization, writing, editing, involved also in validation and computation.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	50 %	60 %

**[5]** Čermák, J., Kisela, T. Delay-dependent stability switches in fractional differential equations. Communications in Nonlinear Science and Numerical Simulation 79, 1-19 (2019). IF: 4.115, Q1

*I was primarily focused on analysis and proofs, visualization and writing, participated in computation and editing, involved also in conceptualization and validation.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	50 %	30 %

**[6]** Čermák, J., Kisela, T., Došlá, Z. Fractional differential equations with a constant delay: Stability and asymptotics of solutions. Applied Mathematics and Computation 298, 336-350 (2017). IF: 2.300, Q1

*I was primarily focused on analysis and proofs, computational work, visualization and writing, participated in editing, involved also in problem formulation and validation.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	45 %	20 %

**[7]** Čermák, J., Horníček, J., Kisela, T. Stability regions for fractional differential systems with a time delay. Communications in Nonlinear Science and Numerical Simulation 31, 108-123 (2016). IF: 2.784, Q1

*I was primarily focused on computational work, analysis and proofs, participated in visualization and writing, involved also in problem formulation and editing.*

Experimental work (%)	Supervision (%)	Manuscript (%)	Research direction (%)
-	-	45 %	30 %