

## Referee's report on the habilitation thesis

**Masaryk University**

**Faculty** of Science

**Habilitation field** Mathematics – mathematical analysis

**Applicant** Mgr. Petr Zemánek, Ph.D.

**Affiliation** Masaryk University, Faculty of Science

**Habilitation thesis** Discrete Symplectic Systems and Square Summable Solutions

**Referee** prof. Gerald Teschl

**Affiliation** Faculty of Mathematics, University of Vienna

### Text of the report (the extent according to the referee's opinion)

From the results in the thesis it is clear that Dr. Zemánek has not only made significant contributions to an active and relevant area in mathematics but he also helped to develop and shape this area. In particular, he is now one of the leading experts on discrete symplectic systems. His results were published in recognized international journals and are presented in a well-structured and easy to read manner. For these reasons I believe that the Habilitation should be awarded to Dr. Petr Zemánek. His research as well as other academic attributes make a solid case for this honor.

### Referee's questions (the number of queries according to the referee's opinion)

**1. no questions**

### Conclusion

Habilitation thesis by Mgr. Petr Zemánek "Discrete Symplectic Systems and Square Summable Solutions" meets the standard requirements for habilitation theses in mathematical analysis.

In Vienna, 21.12.2016



(signature)



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Evaluation report on the habilitation thesis  
*Discrete Symplectic Systems and Square Summable Solutions*  
by Petr Zemánek

I know Dr. Zemánek since his PhD and am well familiar with his work. Since earning his PhD he has established himself as an independent researcher as demonstrated by his research accomplishments summarized in the present thesis and to be discussed below. I will use the same identifiers for publications as used in his thesis.

Dr. Zemánek is working in the area of spectral theory for difference operators, in particular for discrete symplectic systems. These systems, which arise in various applications as demonstrated in Section 1.2 of his thesis, are important to study and are the main topic of the present thesis. A nice introduction together with some background material is given in Chapter 1.

The following four chapters are based on results obtained jointly with his PhD advisor Roman Šimon Hilscher. Chapter 2 begins with the study of square integrable solutions of such a system in the case of a linear spectral parameter. There the foundation of a basic Weyl–Titchmarsh theory (description of separated boundary conditions, Weyl solutions, Weyl matrices, Weyl discs, limit point/limit circle classification) are laid. The results were published in [A15]. In the next chapter, based on [A13], these results are extended to the case of arbitrary boundary conditions. In Chapter 4 an invariance result for the limit circle case for two systems is given. This extends a result of Walker and was published in [A19]. Finally, in Chapter 5, which is based on [A17], some of the previous results are extended to the case where the system depends analytically on the spectral parameter.

The last two chapters are based on joint work with Stephen Clark (Missouri S&T) and look again at the case of a linear spectral parameter. In Chapter 6, based on [A18], the problem of definiteness, minimal and maximal relations as well as deficiency indices are investigated. In Chapter 7, based on [A21], all self-adjoint extensions of the minimal relation are characterized. Moreover, a description of the Krein–von Neumann extension and a limit point criterion is given.

Since all of the above results contained in the thesis were obtained in cooperation I also had a brief look at the single authored papers [A11] and [A14] which derive an analog of Rofe-Beketov's formula for symplectic systems and show how even order Sturm–Liouville-type equations on time scales can be written as a time reversed symplectic system.

From studying these results it is clear that Dr. Zemánek has not only made significant contributions to an active and relevant area in mathematics but he also helped to develop and shape this area. In particular, he is now one of the leading experts on discrete symplectic systems. His results were published in recognized international journals and are presented in a well-structured and easy to read manner. For these reasons I believe that the Habilitation should be awarded to Dr. Petr Zemánek. His research as well as other academic attributes make a solid case for this honor.

Sincerely,



Gerald Teschl  
Professor of Mathematics