MASARYKOVA UNIVERZITA

Attachment No. 11:

Habilitation thesis reviewer's report

Masaryk University	
Faculty	Faculty of Informatics
Field of study	Informatics
Applicant	Mgr. Jan Obdržálek, PhD.
Unit	Faculty of Informatics
Habilitation thesis (title)	Graphs, Their Width, and Logic
Reviewer	Prof. Gregory Z. Gutin
Unit	University of London, United Kingdom

Reviewer's report (extent of text up to the reviewer)

see the report attached

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer) ...

see the question attached

Conclusion

The habilitation thesis submitted by Jan Obdržálek entitled "Graphs, Their Width, and Logic" ***meets*** ~~**does not meet**~~ the requirements applicable to habilitation theses in the field of Informatics.

In …/… on 14/08/2017

Reviewer's Report on Habilitation Thesis "Graphs, Their Width, and Logic" by Jan Obdržálek

The thesis covers several fundamental problems in structural and algorithmic graph theory, partially ordered sets, and parity games.

Part 1 of the thesis has six chapters. Chapter 1 is an introduction. In Chapter 2, the author provides preliminaries on graphs, logic on graphs, model checking, parameterized algorithms and complexity, and modal μ-calculus.

Chapter 3 overviews the most important width measures in undirected graphs such as tree-width, clique-width and rank-width. Section 3.3.2 presents main results of [R. Ganian *et al.* (including J. Obdržálek), Lower bounds on the complexity of MSO_1 model-checking, J. Comput. Syst. Sci. 80(1) (2014), 180–194], where it was proved that model-checking on a wide class of vertex-labeled graphs is not even in XP unless the non-uniform Exponential Time Hypothesis fails (which is unlikely). In Section 3.4, the author discusses the use in directed graphs of widths its underlying undirected graphs. As an example of a positive result, he gives his theorem from [J. Obdržálek, Fast μ-calculus model-checking when tree-width is bounded, in: CAV 2003, LNCS 2725 (2003)], where he showed that any parity game (played on digraphs) admits a fixed-parameter algorithm when parametrized by underlying undirected graph treewidth. To illustrate that undirected clique-width or rank-width does not help us to solve digraph problems, the author gives Proposition 3.21, which shows that unless P=NP, there exist MSO_1definable digraph properties that have no XP-time algorithms with respect to undirected clique-width or rank-width.

Chapter 4 discusses digraph width measures. The author was among a group of researchers who tried to introduce digraph width measures which are in par with those for undirected graphs with respect to structural and algorithmic properties. In particular, he was the first to introduce DAG-width in [J. Obdržálek, DAG-width Connectivity measure for directed graphs, in: SODA 2006, 2006, pp 814–821]. In joint paper [D. Berwanger *et al.* (including J. Obdržálek), The DAG-width of directed graphs, J. Combin. Th. Ser. B 102(4) (2012) 900–923], they, in particular, proved that for each k, there is a polynomial-time algorithm, which determines the winner of parity games on all digraphs with DAG-width at most k. However, a clever observation from [R. Ganian *et al.* (including J. Obdržálek), Are there any good digraph width measures? J. Comb. Theory, Ser. B 116 (2016) 250–286] stated as Proposition 4.11 shows that for any directed width which, like DAG-width , is bounded on acyclic digraphs, there are some MSO_1 properties that cannot be decided on classes of digraphs of bounded directed width in XP-time unless P=NP.

To avoid this situation, researchers considered other directed width measures, which are not bounded on acyclic digraphs. In particular Theorem 4.15 of Courcelle, Makowsky and Rotics showed that every $LinEMSO_1$ optimization problem on a digraph D is fixed-parameter tractable for parameter $t + |\psi|$, where t is the directed clique-width of D and ψ is the corresponding MSO_1 formula. In [R. Ganian *et al.*, Digraph width measures in parameterized algorithmics, Discrete Appl. Math. 168 (2014) 88–107], Jan and his co-authors gave a short proof of the above theorem via a reduction of the directed version into the undirected one with vertex labels.

Chapter 5 is devoted to the very important question of the existence of digraph width measures, which are powerful in both structural and algorithmic sense. As shown in Chapter 4, DAG-width and similar measures while having good structural properties, do not have good algorithmic ones as for them we have no efficient model checking algorithms. We have also seen that directed clique-width and bi-rank-width are more suitable candidates for a good digraph width measure, since

MSO_1 model checking can be done efficiently on classes of graphs where one of these parameters is bounded. Unfortunately, clique-width and bi-rankwidth do not possess the nice structural properties common to the various treewidth-like measures, such as being subgraph- or contraction-monotone. So, a natural question arose: Is there a digraph width measure without drawbacks above. In [R. Ganian *et al.*, Are there any good digraph width measures? J. Comb. Theory, Ser. B 116 (2016) 250–286] Jan and his co-authors answered this question negatively by proving several results based on well-known complexity-theoretical assumptions including P$\neq$NP. This means that many well-known approaches in undirected graph algorithms cannot be extended to digraphs.

Chapter 6 is devoted to fist order (FO) logic on dense graphs such as interval graphs and poset digraphs. In [R. Ganian, FO Model Checking of Interval Graphs, Log. Methods Comput. Sci. 11(4) (2015)] Jan and his co-authors shown that for every finite subset L of reals and every FO sentence ψ, there exists an algorithm running in time $O(n \log n)$ that decides whether an input n-vertex L-interval graph G given by its L-representation satisfies ϕ. They also proved that this result cannot be extended to MSO_1 as there model checking is para-NP-hard even on unit interval graphs. In [J. Gajarský, Faster Existential FO Model Checking on Posets, Log. Methods Comput. Sci. 11(4) (2015)] Jan and his co-authors proved the following very interesting result significantly improving that of Bova, Ganian and Szeider: Poset $\exists$-FO-Model Checking is fixed-parameter tractable in the formula size and the width of an input poset.

The second part of the thesis contains ten papers by Jan Obdržálek and by him and his co-authors.

The thesis clearly demonstrates that Jan Obdržálek produced a large number of important and technically-challenging results in a significant number of papers. The results are important as they are in important areas of algorithms and complexity research. The results were published in leading conferences and journals. I am certain that the habilitation thesis meets the requirements applicable to habilitation theses in the field of Informatics.

Professor Gregory Gutin

Reviewer's Questions for the Habilitation Thesis Defence

- Give a short description of modal μ-calculus.
- What is the non-uniform Exponential Time Hypothesis?
- Courcelle, Makowsky and Rotics showed that every LinEMSO$_1$ optimization problem on a digraph D is fixed-parameter tractable for parameter $t + |\psi|$, where t is the directed clique-width of D and ψ is the corresponding MSO$_1$ formula. Could you give a scheme of the short proof of the above theorem via a reduction of the directed version into the undirected one with vertex labels mentioned in the thesis?
- In [R. Ganian, FO Model Checking of Interval Graphs, Log. Methods Comput. Sci. 11(4) (2015)] Jan and his co-authors shown that for every finite subset L of reals and every FO sentence ψ, there exists an algorithm running in time $O(n \log n)$ that decides whether an input n-vertex L-interval graph G given by its L-representation satisfies ϕ. They also proved that this result cannot be extended to MSO$_1$ as there model checking is para-NP-hard even on unit interval graphs. What is the parameter in the last result?

Professor Gregory Gutin