MASARYKOVA UNIVERZITA

Attachment No. 11:

Habilitation thesis reviewer's report

Masaryk University	
Faculty	Faculty of Informatics
Field of study	Informatics
Applicant	Mgr. Jan Obdržálek, PhD.
Unit	Faculty of Informatics
Habilitation thesis (title)	Graphs, Their Width, and Logic
Reviewer	Prof. Stefan Szeider
Unit	Technische Universität Wien, Austria

Reviewer's report (extent of text up to the reviewer)

Dr. Obdržálek's habilitation thesis focuses on a fundamental area of computer science, namely the classification of the complexity of natural and important problems in the realm of graphs and partially ordered sets. This is achieved by providing lower bounds (in the form of hardness results) as well as new algorithms.

The problems targeted by the thesis belong to the family of so-called *model checking problems*: given a formula phi in some logic L and a graph G, decide whether G models phi. Model checking problems are very well studied in the literature and by now have become a cornerstone of the algorithmic theory subfield of computer science. One of the reasons these problems have become so prominent is their ability to easily capture and encode a variety of standard and natural graph problems, and so tractability results for model checking have immediate consequences for a variety of other problems.

Given the generality of model checking problems, it is of course not surprising that the vast majority of such problems are NP-hard on general graphs and other structures; however, the natural question here is which properties of these structures allow us to obtain efficient algorithms for model checking. This idea is formalized through the use of the parameterized complexity framework and the notion of structural parameters.

The habilitation thesis presents results obtained by Dr. Obdržálek (on his own or in collaboration with various coauthors) for the following three problems:

1) Mu-calculus model checking and the closely related parity games problem
   Here, Dr. Obdržálek showcases some of his earliest work, which contributed to establishing the digraph parameter DAG-width and obtaining some of the first tractability results for parity games (which are virtually equivalent to mu-calculus model checking). This is the topic of papers A, B and C in the habilitation. Papers A and B present new algorithms for parity games using the undirected treewidth and directed clique-width, respectively, while paper C establishes the structural parameter DAG-width and showcases its algorithmic applications, most notably including an algorithm for parity games.
2) Model checking monadic second-order (MSO) logic
   This set of results, presented in papers D, E and I, introduces new lower-bound machinery and rules out the existence of efficient model checking algorithms for MSO logic in certain settings. Paper D extends earlier lower-bound results of Kreutzer and Tazari to a weaker variant of MSO (MSO_1). Paper E then shifts its attention to directed graphs, and shows that a number of natural problems (including, notably, MSO model checking)

remain difficult even when natural directed parameters such as DAG-width and Kelly-width are bounded. Paper I then extends the results of paper E and rules out the use of any inherently directed graph parameter, defined by a set of basic conditions, for efficient model checking of MSO logic.

3) Model checking first order logic

The final set of results, presented in papers F, G, H and J, focuses on a less expressive logic, notably the classical first order logic. The first results presented by Dr. Obdržálek in paper F identify the conditions under which first order logic can be efficiently model checked on interval graphs. Paper G then shifts the focus from graphs to partially ordered sets (posets), and improves an earlier result of Bova, Ganian and me by presenting a more efficient algorithm for model checking existential first order logic on posets of small width. Paper H solves an open question of ours by establishing the fixed-parameter tractability of FO model checking on posets parameterized by the width, and is perhaps the most technically challenging result of the whole habilitation thesis. Finally, paper J loosely builds on the ideas and results of paper H by studying the complexity of FO model checking on dense graph classes: the primal graphs of partially ordered sets (viewed as a relational structure) are nearly always very dense, and it was natural to ask whether the new model checking techniques used to handle posets of bounded width could be extended to other dense graph classes. Paper J identified a set of general properties which are sufficient for efficient first order model checking on dense graph classes.

All in all, the contributions presented in the habilitation thesis are significant and have helped considerably expand our knowledge of the complexity of model checking. Several of the papers included in the habilitation thesis appeared in top computer science venues such as FOCS, LICS, the Journal of Computer and System Sciences and the Journal of Combinatorial Theory series B. The thesis also includes some of Dr. Obdržálek's earlier single-author papers, and one may reasonably assume that his contribution to the joint-author papers was not marginal. Overall, this presents a very strong case for the habilitation of Dr. Obdržálek.

Typos and minor corrections (to be fixed in the electronic version):
-page 3, "what is the fastest a combination of at most three connecting flights…":
Remove "a"

-page 11, "are actually optimization"
Change to "are often optimization"

-page 15, "fact, that"
Delete comma

-page 26, "Not to further duplicate our efforts we decided to produced"
Delete the last "d"

-page 36, "(Arc is …)"
Add "An" in front of arc

Reviewer's questions for the habilitation thesis defence (number of questions up to the reviewer)

- Aside from the width of posets, another very frequently used structural measure in this area is the notion of *poset dimension*. In particular, poset dimension is a more "general" measure than width: the width can be used to upper-bound the dimension, but the dimension can be arbitrarily smaller than the width. While computing the poset dimension is NP-hard, it is still a natural question to ask whether one may obtain fixed-parameter algorithms for model checking of FO logic (or its fragments) parameterized by poset dimension if a witness (decomposition) for the dimension is provided as part of the input. Are you aware of any lower bounds which would rule out such a result?

- Both your and Fearnley, Schewe's algorithms for solving parity games using undirected treewidth represent XP algorithms when the number of priorities is considered to be part of the input. Has any work been done on improving these to fixed-parameter algorithms parameterized by treewidth, i.e., algorithms of the form poly(n+d) times f(k)? One cannot hope to show lower bounds here due to the fact that the classical complexity of parity games remains open, but an algorithmic result like this should be much easier to obtain than a (hypothetical) polynomial-time algorithm for parity games.

Conclusion

The habilitation thesis submitted by Jan Obdržálek entitled "Graphs, Their Width, and Logic" ***meets*** the requirements applicable to habilitation theses in the field of Informatics.

In Vienna, Austria, on 23 October 2017